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CABINETMAKERS’ WORKPLACE MATHEMATICS AND PROBLEM SOLVING

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Abstract

This study explored what kind of mathematics is needed in cabinetmakers’ everyday work and how problem solving is intertwined in it. The informants of the study were four Finnish cabinetmakers and the data consisted of workshop observations, interviews, photos, pictures and sketches made by the participants during the interviews. The data was analysed using different qualitative techniques.

Even though the participants identified many areas of mathematics that could be used in their daily work, they used mathematics only if they were able to. The cabinetmakers’ different mathematical skills and knowledge were put to the limit.

Cabinetmakers were found to constantly face problem solving situations along with the creative processes. Being able to use more advanced mathematics helped them to solve those problems more efficiently, without wasting time and materials. Based on the findings, the paper discusses the similarities and differences between problem solving and creative process. It is suggested that the combination of craftsmanship, creativity, and efficient problem solving skills together with more than basic mathematical knowledge will help cabinetmakers in adapting and surviving in the future unstable labour markets.

Keywords

Workplace mathematics, problem solving, creative process, jigs, cabinetmakers

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Introduction

Working life and the needed know-how at workplaces is changing all the time, not least because of technological advances. These change processes, along with the constant demands of efficiency, questions what kind of skills and knowledge are needed to manage or succeed in working life, and what should be taught in vocational education. Should the focus be just on the context-bound knowledge needed at a specific work? From the point of efficiency, it would sound reasonable. However, there are scholars (e.g. FitzSimons 2014) who remind us that the only significant issue at work is the constant need to learn things and solve problems that do not yet exist and for which we do not have any prior experiences. To be able to solve that kind of problem professionals must produce and use new kinds of knowledge and reproduce the old ones. According to FitzSimons, many such problems need creative and innovative solutions where mathematical knowledge has a significant role. That is why research should focus more on how various emerging and even unexpected problems are solved in workplaces and on exploring what kind of mathematical knowledge is activated in those processes.

In this study, the aim is to find out what kind of mathematics is needed in cabinetmakers' everyday work and how problem solving and finding solutions to emergent problems are intertwined in it. The cabinetmakers' profession is located interestingly between the old, traditional handcraft methods and new technology. According to Ministry of Economic Affairs and Employment (2015; Tuomaala 2016; www. ammattibarometri.fi) in Finland the cabinetmakers' profession is one of those that are at great risk of unemployment in the near future and this fact does not seem to be unique elsewhere (Frey & Osborne, 2017). Being an important part of the Finnish wood industry, mass-produced furniture is the outcome of business expertise and engineering skills combined. At present, modern wood factories employ all sorts of specialized workers in the different product elaboration phases (e.g. assemblers, machinists, hand-sanders, finishers). However, the basics of furniture production will always be a craft-based industry due to the use of a natural material. Prototype-work is in any case based in craftsmanship. Hence, some cabinetmakers will still be needed in the future (Publications of the Ministry of Education and Culture, Finland 2017:17). But who will survive in the future unstable circumstances?

This paper is structured in the following way. First the main research done in the field of workplace mathematics, problem solving and creativity is outlined. Second, the research questions are brought to light and the methodology of the work is described. Third, the different circumstances of each of the participants are revealed and the paper continues with a cross analysis of the core subject matters -i.e., mathematics in use, problem solving and creative process. Last and based on the findings, the similarities and differences between problem solving and creative processes are discussed.

Workplace mathematics under consideration

Both mathematics and workplace are terms embracing profound crucial interpretations of their meaning and effect. The workplace is the site at which a person produces work and it might be located in any place where work is performed including homes, offices, manufacturing facilities, farms, stores, workshops or outdoors. Workplaces have existed for a

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long time and they will perpetuate in the future but modified and adapted to the moment, as it has happened until now. Therefore, the different settings, practices and dynamics embedded in the workplace have been of great interest in different research fields (Arminen, 2001; Virolainen, 2007; Pajarinen, Rouvinen & Ekeland, 2015). On the other hand, mathematics is known as well to be everywhere around us. Accordingly, more and more studies have been driven to inspect the influence and impact of mathematics outside the formal setting of the school and consequently, in the workplace (Moreira & Pardal, 2012; Saló i Nevado, Holm & Pehkonen, 2011; Zevenberger & Zevenberger, 2009).

The combination of both concepts generates an attractive fundamental outcome and it is the reason why, at least over the past two decades, researchers have been keen on studying different workplace settings and the mathematical concepts and processes used by different professionals. For example, Pozzi, Noss and Hoyles (1998) studied paediatric ward nurses dealing with ratio and proportion problems and discussed the implications of workplace practices and emphasized how valuable are the informal strategies used in the ward. Also, Saló i Nevado, Holm and Pehkonen (2011) explored how farmers dealt with distributing the space in a barn to feed calves and how they used different items as measuring devices. Their study reassured the significance of spatial sense and how basic numeracy allowed the farmers to succeed in rather complex context-specific situations.

Earlier, Millroy (1992) focused on the carpenters' geometric ideas and strategies, and ratified the tacit mathematical knowledge in the carpenter's actions. In her study, the mathematics at work were considered from the point of view of the participants and she documented mathematical concepts and processes such as spatial visualization, proportionality or symmetry. In another ethnographic study Moreira & Pardal (2012) examined masons' professional practices in Portugal aiming to illustrate the mathematics embedded in the daily practices of the masons. Their work described in detail how geometry and arithmetic emerge from the masons' work tasks.

Some researchers have attempted to view the mathematical practices at work through the eyes of school mathematics. An example of this is the project of Hogan & Morony (2000) where teachers were sent to find mathematics in different workplaces. The study gathered their reflections on different aspects of the research such as the impact on their thinking, doing research and mathematics in the workplace. The teachers were sent to the workplaces, shadowed workers for one day, conducted an interview and wrote about their findings (2000, 101). The project revealed that the teachers were taken aback by the amount of mathematics found in the workplaces and the mathematical skills displayed by the workers. Bessot (2000) questioned whether it is admissible in teaching to transfer mathematical knowledge that has been shaped and altered at the workplace. She looked into how construction builders constructed temporary moulds to build a wall on an inclined slab and contrasted the mathematical knowledge used to that transmitted by teachers in high school. She alleged that in a construction site there are further considerations to be made before using something mathematically. She mentioned two aspects: one is 'the anticipation' of the actions to be used and the second one is the 'verification of the result of the actions' used. These two aspects are not always self-evident in the mathematics taught in French high schools, since they are not visibly needed for the students. Teachers are aiming at the practical use of the mathematical knowledge. Often the conditions of the reality, where the mathematical knowledge might be used, do not allow such applications.

Magajna & Monaghan (2003) used Saxe's four-parameter model (Saxe, 1991) to examine the work practices of glass factory technicians. Their study resolved that while good understanding of mathematical concepts is often required, most significant is to be able to relate the mathematics to the context (2003, 121). Saxe's model was developed to elucidate mathematical practices in a cultural transition and it was focused on emerging goals under four parameters as in activity structures, social interactions, prior understandings and conventions and artefacts. Saxe applied the model in studies of street-sellers' practices.

In terms of mathematical content, there are studies that claim a constant appearance of mathematical elements such as proportionality, approximation, basic geometry, etc. (Greiffenhagen & Sharrock, 2008) and not only basic arithmetic (Williams & Wake, 2007; Straesser, 2000) or simple algorithms (Riall & Burghes, 2000; Hoyles et al., 2001). Thus, it is clear that mathematics is embedded in countless diverse workplaces. However, up to certain extent what early studies before the 1990s seem to disregard is that mathematics is much more than the use of arithmetic or geometry (Cockcroft-report 1982). Mostly the studies mentioned up to this point have looked at the specific mathematical knowledge and some of the mathematical practices. In other words, as the literature review shows, researchers in this field have shed light on various practices, mathematical concepts, contents and tools that are embedded in different professions. To some extent, previous studies show how school mathematics and workplace mathematics differ from each other; even though, one of the primary goals of mathematics teaching and learning is to develop the ability to solve a wide variety of complex mathematics problems that may occur at the workplace (Stanic & Kilpatrick, 1988).

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FitzSimons (2014) asks what actually is vocational or workplace mathematics. According to her in todays' context of globalization and rapid technological, social, economic and environmental changes, the most or even only significant issue is the constant need to learn things that do not currently exist, and to solve unexpected problems for which there are no any prior experiences. In order to solve future problems, one must be able to produce and use new forms of knowledge and re-contextualize the old, existing ones. These kinds of problems are likely requiring creative, innovative solutions, where mathematical knowledge has critical role to play. That is why, research should focus more on how people find solutions to various, even unexpected problems that emerge in workplaces and to explore what kind of mathematical knowledge is activated in those processes.

Problem Solving and Creativity

A problem is by definition something that one does not have the experience to solve (Resnick and Glaser, 1976) or when a person has a given aim, but he/she does not know how to reach it (Duncker, 1945). Accordingly, Mayer (1990) defines problem solving as the collection of the cognitive processes that take place when transitioning from the current state where one does not know what to do to the final state where a solution is found (as cited in Csapó & Funke, 2017, p.62).

Correspondingly, when past experiences are enough for dealing with a *problem*, it cannot be considered a *problem* and it becomes an *exercise* or a *task* (Liljedahl, 2004). When solving a *problem*, one makes use of past experiences in addition to direct efforts and a sudden inspiration what Hadamard (1945) would call illumination in the creative process. It is at this point where the *problem solving process* and the *creative process* get intertwined and, up to some extent, fused (Van Harpen & Sriraman, 2013). In many studies, the distinction has not always been obvious and, for example, in studies entirely aimed at creativity, the participant's account of the creative work process is labelled as "an open-ended process without a clear direction to an end" with an unlimited time commitment (Taylor, 2012, 49), which is basically a definition of problem solving. Also, other studies use the terms as if they were synonyms (see for example Lubart, 2001). Nonetheless, for this study it is an imperative to separate and differentiate these two concepts as Wimmer (2016), who in her short essee argues that "successful problem-solving can be regarded as a sufficient condition of the creative process"

According to Liljedahl and Allen (2013), the different understandings of what problem solving is may be summarized in six divergent lines. The first one is problem solving by design, where prior knowledge and experience shape the process of the problem solver and infer the chosen strategies (Bruner, 1964). The second line is Pólya's Heuristics (1957) and the four stages of problem solving: understanding the problem, conceiving a plan, executing it and reflecting over it. Up to certain extent, this line is a polished version of the problem solving by design, since in order to succeed in the four stages, once again one must rely on prior knowledge and experience. In the third place, Alan Schoenfeld distinguished different strategies that individuals use spontaneously (Schoenfeld, 1983). He defined problem solving as a process where an individual's prior knowledge, actions and views collide, emerging within a certain context. Fourth, is Perkins' "breakthrough thinking" (2000) where problem solving is a process that depends on extra-logical kick that he calls "breakthrough thinking". In this process, the individual must first admit being stuck without a strategy and proceed to what he calls introspection. Fifth, Mason, Burton and Stacey (1982) present another line in "thinking mathematically". For Mason et al., problem solving involves the processes of specializing and generalizing. Specializing is presented as a phase in which the individual gets to know the problem per se. Generalizing is understood as the part of the process when solutions are tested. According to Liljedahl and Allen (2013), the sixth and last line in problem solving is the gestalt psychology of problem solving, which defends that problem solving cannot be taught since it is a product of insight (Koestler, 1964) and that a problem may be solved by turning it upside down over and over (Liljedahl & Sriraman, 2006). The main criticism of the gestalt's vision is that the inside moment is unattainable and cannot be researched.

On the other hand, the conceptual framework of the creative process emerged from Wallas (1926). His model was linear and had four stages: preparation, incubation, illumination and verification. Hadamard (1945) redefined Wallas's model while working on conceptualizing the process of mathematical invention (see Sriraman (2004) for other creativity models) and transformed it into a stage theory (Liljedahl, 2009). For Hadamard, Wallas' stages embraced the whole process of creation including unconscious phases. Initiation is the stage where the first consciously intended work takes place. It can be regarded as the first encounter with the problem and where the setting is compared with past experiences while searching for a solution (Bruner, 1964). In the second stage, regarded as the incubation stage, the person stops working on it at a conscious level (Poincaré, 1952). The third stage is the illumination stage where the unconscious bonds with the conscious in a brisk of lucidity of a possible solution. Liljedahl (2004) regards it as the "AHA! Experience". Verification is the fourth and final stage where the suitability of this emergent idea is evaluated. In this article, Hadamart's redefined creative process model of Wallas linear one is used as a broad frame to start the analysis.

Research Questions

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This study starts with the assumption that it is not possible to work as a cabinetmaker without some mathematical knowledge. In order to reach the aim of finding out what kind of mathematics is needed in cabinetmakers' everyday work, the following question is posed: What are the mathematics in use needed by cabinetmakers? To answer this question, the study first explores the mathematics needed in everyday work that is identified and labelled as mathematics. Since this question is mathematics-based, the study looks at work through the “lenses of mathematics” from both perspectives, as a participant and as an outsider.

Accordingly, to reach the aim of finding out how problem solving is intertwined in cabinetmakers everyday work, the following questions are posed: What are the typical problem solving situations faced by cabinetmakers and how does the problem solving process proceeds? To answer these questions, problem solving situations faced in cabinetmakers’ work are considered. By them, the study refers to the challenging, problematic situations in the work process, which must be solved and need solutions and acts to proceed to the next stage. The starting point is the work itself, the problems emerged and how the cabinetmakers find solutions to various, new, even unexpected problems that emerge in their workplace.

Methodology

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The participants and their workplaces

The informants of this study are four Finnish cabinetmakers and their workshops represent the context of the workplace. The three different workshops were located in the metropolitan area of Helsinki in Finland. One of the workshops was situated in a vocational school and it was used for the teaching purposes as well. The workshop was well equipped and had modern machinery. The second workshop was a reformed old farrier workshop with traditional and old-fashioned machinery as well as modern. Several craftsmen used this workshop during their free time and for personal projects. The third workshop was a rented space from a warehouse where several cabinetmakers and companies had workshops. Here different tailor-made furniture was produced.

All four participants were Finnish male cabinetmakers, from 38 to 65 years old, with the same vocational school training. In the Finnish educational context, it means that they have studied mathematics a minimum nine credits out of a total 180 (Finnish National Board of Education, 2013; Opetushallitus, 2016). Each of the participants had experience in the labour markets. They either had their own company or worked for someone else. All of them were respected and skilled craftsmen in their field. For the research purposes the participants were named Jacob, Thomas, Anthony, and Frank.

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Data Collection

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An ethnographic approach was used in the data collection, which has been pointed out to be an appropriate methodology when trying to understand mathematics from participants’ point of view (Barton, 1997; Hodson, 2004; Atkinson & Delamont, 2005). The main data consisted of workshop observations with fieldnotes, interviews, videos and photos. In addition, the data was completed with pictures and sketches made by the participants during the interviews. The data was collected in three phases.

During Phase I, the data was gathered via workshop observations and several individual semi-structured interviews of each participant (Rapley, 2001; Atkinson and Delamont, 2005), where the cabinetmakers were asked to describe in detail their daily routines and tasks at work. They were invited to have the first interview with open questions in anticipation to guide the conversation such as *'Please, describe an average day at work' (to get an overall description of a typical day), 'what do you do when you get here?' (to get a more detailed list of actions and happenings upon arrival to the workshop)*. The sites were visited several times. Each cabinetmaker decided the place to be interviewed and, except for the first interview with Jacob, which took place in his own house, the rest of the interviews were conducted in the respective workshops of the cabinetmakers. All the interviews were audio recorded and later transcribed. In the interviews, some questions were made to elicit detailed descriptions of the cabinetmakers daily routines and tasks: first some general questions about the cabinetmakers background and education in the field and gradually more exhaustive questions about their tasks and details of their job (such as '*what do you do when you get a client contacting you? Please walk me through each and every step?', 'What do you do when you deal with something else than 90 degrees angles in a piece? Can you show me?'*).

Phase II of data collection took place after the initial analysis of the data collected in the phase I. Its aim was to focus particularly on how the cabinetmakers conceived the problem solving situations. During the initial analysis of the Phase I data, it was found that "making jigs" was a typical problem solving situation in the participants' everyday work. Therefore, the phase II had a targeted approach since it was needed to better understand these situations. In this phase, the cabinetmakers were asked to show different types and examples of jigs and explain their uses.

PICTURE 1 APROX HERE!

Picture1: two jigs in one board (numbered) for making the arms of a trivet. In jig number two it can be seen how is the piece of wood fastened.

PICTURE 2 APROX HERE!

Picture 2: Jig to guide the router when making a hole. In the picture, the router is being guided by the jig (wooden plank with a hole)

Jigs are self-constructed appliances for guiding the machinery or supporting the assembly in a specific stage of the job (Paavola & Ilonen 1981). In other words, jigs are aids in the working process and typically needed for a unique situation. The informants were asked about the process of creating those jigs. Since each jig is related to a project process, several projects were pursued, for example: a tool closet door, a trivet, a wooden sandal, a wardrobe or decorative wooden icosahedra. All the jigs needed to build a four-sided trivet and a pentagonal trivet, were discussed. Field notes, researcher reflections and memos were collected during the observations. In addition some photographs were taken and videos were recorded mainly collected to support the interview data.

Additionally, the Phase III of data collection was meant to document a cabinetmakers project from the very beginning until the end, to see the spontaneous appearance of problem solving situations. Jacob was asked to take part in the project documentary where he was to contact the researcher every time he was going to advance in the development of the project. The data was video-recorded and the shadowing interviews (Blake & Stalberg, 2009; Quinlan, 2008) were unstructured with open-ended questions such as *Could you tell me what you just did?* or *Can you put in words what you just did to create that ellipsis?*. They aimed to obtain descriptive data of the cabinetmakers' performance. The video-recorded data

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and the photographs helped to recognize the mathematics that the cabinetmakers were not able to see by themselves. For this piece of the study, the Phase III data provided additional and more sharpening data of the jig making.

Data Analysis

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In the beginning, the collected interview data were transcribed and an inductive qualitative data-analysis (see Thomas, 2006) was applied. It meant detailed reading at the raw data of and - in this case - also looking at it. The derived concepts and themes emerged from the data

Hence, the analysis started by close reading of the interview data and and fieldnotes from observations, writing memos and summary sheets and coding the photos and so that they could be connected to the other data.. Identifying the emergent themes was the next preliminary phase in the analysis. The themes were used to reach preliminary order in the data and to help understanding of the cabinetmakers' work and to advice what type of data was needed to obtain in the following phases. The identified nine themes (work strategies, daily routines, technology, experience and skills, feelings, tools, working processes, problems and materials) emerged in all the interviews. The cabinetmakers explained thoroughly their daily tasks, how their day was organized and what type of jobs they had to do. A big part of their descriptions ended up as examples of how to use tools and technology and how to handle materials for optimizing the results. When describing the working processes, they explained accurately all the stages of their job starting from the point when a customer makes an order. Along with these descriptions, feelings and experience of past projects were manifested. The data indicated that problem solving had an inevitable role in cabinetmakers' everyday work. Typically, the problem solving emerged in a situation when a needed jig was to be constructed. Since the jigs typically are unique tools they need to be "invented" in the construction process. So, it was an imperative to consider the linkages between problem solving and creativeness, and to obtain more new data about problem-solving.

The analysis continued during and after the data collection by ordering the thematised data under the topics of *mathematics*, *problem solving* and *jig creation*. The *mathematics* was further analysed thematically identifying various use of mathematics in the cabinetmakers daily work. Through the "lenses of mathematics" the use of basic calculations, precentage, measurement, estimation, geometry and trigonometry could be derived from the data. Some of the use of mathematics the cabinetmakers were able to identify and label by themselves as mathematics, some could only be depicted from the observations, fieldnotes and videodata. In addition, the different mathematical skills and knowledge of cabinetmakers became evident. Collaborative data analysis (Cornish, Gillespie & Zittoun, 2014) was applied through the entire analysis process. Concerning *problem-solving* and *jig creation* the significance was attached to the different stages of the processes and by refining the understanding of the cabinetmakers' procedures. To sketch the problem-solving and jig creation procedures a huge poster-type sheet was produced with the data from observations, fieldnotes and photos linked to the interview data. The collected video-data was systematized by extracting different excerpts that deepened the different notions that appeared in the cabinetmakers' interviews. The researchers read carefully ordered data, made first independent interpretations and later on discussed them to reach shared understanding and interpretations. In addition, the cabinetmakers were given the opportunity to consider the interpretations, and some minor modifications were made

Contextualizing the research

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With-the-grain approach

In the world of cabinetmakers, when working with wood, ‘With-the-grain’ cuts are done on the wood parallel to the long axis to expose plain grain. In this paper, the title “with-the-grain approach” is used as a metaphor, since the aim is to ‘expose’ and describe the different cabinetmakers’ settings at the time of data collection.

Jacob

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Jacob was 38 years old. He had his own workshop but he was not working as a fulltime cabinetmaker. He had over 20 years of experience in the field and made tailor-made pieces and chose his customers according to the time and the amount of work. He worked in the old farrier workshop that he had reformed and adapted for his needs as a cabinetmaker. Jacob considered himself to be a traditional cabinetmaker and he used the term “old fashioned” to explain that he liked to work with the timber the old way, without computer blueprints and making joints without screws or nails. Jacob enjoyed working with his hands and liked to feel the wood, establishing a dialogue with the different timber he used. He loved to touch and manipulate the pieces in his hands while he got lost in thought. When Jacob got excited about something, he took pleasure in thinking about it over and over and maturing the idea for a long time before taking action. This made him a perfectionist and resulted in using a lot of time in his projects, always finding room for improvements. Jacob did not easily give up and he tried and tried repeatedly until obtaining the desired result. Regarding collaboration with other cabinetmakers, Jacob kept a small intimate circle and shared his ideas only at one-to-one level.

Thomas

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Thomas was 47 years old and he had almost 30 years of experience as a cabinetmaker. He ran his own company with several workers and his workshop was a rented space in a warehouse where other cabinetmaker firms were located. In his workshop and through the years, he had been collecting diverse tools and machines, which he considered to be life-long investments, particularly jigs of different past projects. Thomas had a vast experience as a cabinetmaker and described himself as traditional in his methods and ways to work. He claimed to love mathematics but he refused to use advanced mathematics (i.e. trigonometry) and computers in his daily tasks. According to him, basic mathematics in addition to trial and error repetitions did the job. He was social and sought human contact while working; for example, he valued the coffee breaks outside the workshop engaging in conversation with other cabinetmakers. Thomas described those as moments for thinking and, for him, spending time thinking about something was a crucial element in any process. Thomas often had apprentices at the workshop from different vocational schools. He liked to pose problems for them, for example, give the apprentice a model of a perfect wooden icosahedron and ask him to replicate it. The apprentice could spend several days or even weeks looking for a way to do it. Thomas claimed that learning by doing was the most important thing to build up a good foundation of experiences for the future and when experience would fail, a conversation with others may enlighten some sort of solution.

Anthony

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At the time of the data collection Anthony was 40 years old and he was working as a cabinetmaker teacher in a vocational school. He had been in the field for almost 20 years and he used the workshop of the vocational school to work for his own projects outside his working hours. Anthony was serene and patient while working. He was keen on experimenting with other materials such as metals and he fully relied on and used different computer based machines. He knew how to use mathematical knowledge and he applied it every time he had a chance in order to be more effective and exact. He was able to use different computer programs and the blueprints for the jobs. Anthony was happy to explain and share the knowledge and reasoning behind his actions and at all times he seemed to be able to link it to the mathematics behind each procedure and tool. He was curious at times, a good observer and at the same time eager to start a conversation about the pros and cons of a detail.

Frank

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Frank was the oldest of all. He was over 60 years old at the time of the interview, but he had finished his studies as a cabinetmaker recently. He had more than 5 years of experience. Frank was using both the vocational school workshop for bigger projects and Jacob's old farrier workshop for smaller ones. Frank was unperturbed, quiet and reflexive. He was not too keen on discussions but enjoyed a friendly talk with a colleague. Often Frank wanted to check his procedures with other more experienced cabinetmakers. He was rather traditional in his taste but reluctant to do things he was not acquainted with. He did not utilize advanced technological machinery and blueprints since he felt insecure operating them. In other words, he preferred to use secure and well known procedures rather than to risk using unfamiliar methods regardless of the perfection of the outcome.

Across the grain approach

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In the world of cabinetmakers, when working with wood, a cut across the face of a board will reveal end grain. Likewise, this section is named "across-the-grain approach" as a metaphor since the research pulls from across the settings of the four cabinetmakers themes pertaining to problem solving at the workplace and therefore revealing the findings.

Mathematical knowledge in use

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The idea behind this study was not to claim an innovative mathematical behaviour of the cabinetmakers, nor to discover a new use of mathematics. It begins with assumption that cabinetmakers used mathematics (Milroy, 1992; Greiffenhagen & Sharrock, 2008). The data supported this initial premise. In the interviews the cabinetmakers described and identified the possible mathematics in their work. It also was depicted from workshop observations. In the following quotation Anthony listed the possible mathematics faced in the everyday working situations:

"Of course (I need mathematics), when I estimate the price for the customer, I must use adding, subtracting, multiplying, dividing and also percentages… I would have to use percentages… and I usually work with fractions and then when I plan the work, I would have to use some geometry…. Also work with trigonometric functions and the percentage again… also when I work with the finishing materials, different kind of… you know, paints and stuff… then I'll have to estimate percentages and amounts and when I use pressuring tools, I have to count pressure… which is mainly multiplying…" (Anthony)

In the daily tasks, all the participants identified the constant use of basic operations such as addition, subtraction, multiplication and division. These were needed for example when measuring pieces, cutting boards, assembling, gluing, making joints and even hole drilling. However, because of the different data types it was possible to depict also the use of mathematics that was not identified by the cabinetmakers. The measurement, which is an essential mathematical dimension of cabinetmaking, could be depicted only vaguely from the interview data. On the other hand, it became clearly visible in the observation data. For example, one of the videos showed Anthony describing the process of making a dovetail joint and all the measurements he needed to consider during the process. Another video recorded Jacob showing how he would measure where to drill the hole to install the leg of a table. In both cases, basic operations dominated their descriptions. Most of the time, the measurements needed to be transformed and operated on before being used in the next step. Interestingly, the participants seem not to identify it as "mathematics". All the participants claimed that the simple basic mathematics is sufficient in everyday work, if they did not face situations that require breaking the routines. "*I think they are very basic operations. Like if I get the le…If I know the maximum length, the main measure and I know that the front frame should be… 30 millimetres shorter from both ends. I must make a subtraction. Multiplication and division and that is really enough" (Jacob).*

When Anthony was asked about the mathematics of making joints, which is an important operation at their work, he replied: *"It is kind of easy math actually, mainly subtracting and adding"*

When estimating prices and taxes, the participants needed to count percentages. These were needed also for calculating the amount of various substances to be used when finalizing the pieces.

"*When I work with the finishing materials, different kind of... you know, paints and stuff... then I'll have to count percentages and estimate amounts"* (Antony).

All four cabinetmakers told about and showed the use of proportions when doing, for example, the measurement drawings of a piece to get a glimpse of how the different parts look when assembled or for dimensioning it. Proportions were also used for proportional reductions or piece enlargements. For example, Jacob mentioned using proportions when constructing a miniature prototype of a piece and when doing its isometric projection. This was particularly seen in the video recordings of Jacob in his workshop.

The skill to estimate quantities and times was very essential to the self-employed cabinetmakers. They must be able to estimate time required to complete the project including the preliminary preparations of the materials (for example the drying the wood), estimate the real prices for handmade products, and needed amounts of materials and components and storing them. Good estimation skills save time, money and materials and they protect the cabinetmakers from making fatal mistakes. Estimation can be quite complicated and exceeds the limits of pure mathematical estimation. Self-employed cabinetmakers must hang onto their old customers, find new ones and consider the consequences of their own actions. Jacob discussed: *"I also have to contemplate how much I want to do this project, because if I realize that this furniture will cost so much that that customer will not ever, ever, never buy it. I can…It's somehow it's mathematics. I have to decide if this is an important way to make a new contact. And if I get this new contact… can I estimate the right prices after this project and get this back somehow. That's the one and ...actually it's the most important thing, because cabinetmaker companies are small and make unique stuff"*

The cabinetmakers draw a great deal of outlines and working drawings both for their customers and for themselves. In them and in perspective projections of the pieces they needed plane and 3D geometry. Geometry is very important in many other ways, too. The cabinetmakers calculated areas, diameters, perimeters, volumes and various transformations of them. For this study, an interesting detail was that the amount of timber was often calculated in litres, to avoid decimals. Measuring and calculating the angles were needed – at least in principle – in planning and drawing joints and final pieces, as well as for adjusting the blades of the saw to the needed cutting positions. Here, the cabinetmakers' mathematical skills and knowledge are put to the limit. Where others insisted that trigonometry was not essential, one of them could apply trigonometry and found it absolutely essential in his work.

"We are using trigonometry all the time. It is our alpha and omega. You will always end up in trigonometry" (Anthony)

When one of the cabinetmakers, who claimed not to be so keen on using trigonometry, was asked how he was able to make any other angles than 90° or 45° without trigonometry, he replied*: "you can do it by trial and error, you know, but it can be kind of… it is really you would use a lot of material and a lot of time… because you actually will have to make a 1 to 1 size model to see that actually work"(Anthony)* When this cabinetmaker faced problems, he would to turn to his colleagues or to a professional (mathematics) textbook for help. In the course of the interview the researcher and the participant were looking at the textbook in question and searching for the formula of the adequate trigonometric function. Then it came out that the participant had no clear idea what he was searching for. He admitted that he would benefit from better trigonometric skills and knowledge:

(P) "If I'm in the workshop and have hundreds of pieces, you really can't make 'test-assemblies'...

(I): For every single piece. You need to...

(P) You have to count and then comes the really, really big problems if you can't do that."

In an interview Thomas discussed the upper limit of the need for mathematics: *"Quite seldom… sometimes we … we just had an affair with ellipses…. we ended up with an equation of second or third degree. But very, very seldom and it is only just if you are interested in taking that kind of jobs. So, the trigonometry is sufficient for cabinetmakers. But, of course also in trigonometry … it depends, how you are involved in it. If you want to calculate angles of miter joints in various pyramids, you can get really hard equations. Then, involuntarily you will end up to equations of second degree. When you have two variables, you cannot avoid it. But, there are not many cabinetmakers who will bother their head with so difficult mathematics".*

All the cabinetmakers in the study identified and used mathematics in they work. However, the findings suggest that it is possible to manage with quite elementary mathematics, even when the cabinetmakers have succeeded in their careers. As Thomas put it, it is a matter if you are *"interested in taking that kind of job"* which in order to be completed require more advanced mathematics - or alternatively - a lot of risk-bearing experimenting. Hence, besides and instead of applying advanced mathematics they turned to slow and resource consuming trial and error –methods. On one hand, the study can try to find some explanation from the participants' different mathematical skills and knowledge. On the other hand, the explanations may lie in the fact that the properties of wood do not work fully in the ideal world of mathematics. That is why, sometimes all cabinetmakers had to accept experimenting – despite of their knowledge of mathematics: *"Wood is wood... and it not always so precise. And if you just count, there remains a hole between the pieces, and you shouldn't let that happen... it is of a better quality if the pieces are together. If you compare that you are very good with trigonometry...you can use it very well, but for some reason it doesn't match. It's more important that the pieces are together." (Jacob)*

Problem solving at work

During the process of data collection and preliminary analysis of the Phase I data, from time to time the cabinetmakers faced problematic situations and operations where they did not immediately know how to act and did not have routine solutions. They also recognized these situations themselves. During the interviews, Jacob was reflecting on his work and

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defined unintentionally the term problem: *"There is also very often that type of project that you are doing something for the first time and you don't know (how to proceed)"*. This is very near to Hayes' (1980) understanding of the term "problem" according to whom a problem is the whole between the present stage and the final goal, when the steps to follow are unknown. From this viewpoint, Bodner (1987) suggested that if the steps are known, the whole setting becomes a task, whereas if they are not known, the setting turns into a problem solving situation. Schoenfeld suggests "*a problem is only a problem if you don't know how to go about solving it. A problem that has no 'surprises' in store, and can be solved comfortably by routine or familiar procedures (no matter how difficult!) is an exercise"* (1983, 41).

Problem solving seemed to be very natural in their daily tasks, therefore inevitable and intrinsic. As Anthony put it *"problem solving is a very essential part of my work as a cabinetmaker. A great deal of my work tasks can be described as problem solving, starting with the customers' needs and ending with post-delivery issues. The most central problem in all designing and manufacturing is integrating outer appearance, functionality and costs"*. However, problem solving was not referred as simply a fragment of the cabinetmakers daily routines. The participants regarded problem solving often as the most difficult stage of the process when an unknown procedure needed to be done in order to proceed to the next step of a known process.

Particularly interesting were the situations when the cabinetmakers had to plan and construct "the needed new jig". Jigs are self-constructed and typically unique custom-made tools needed constantly in cabinetmaking. To plan and make the jigs involve many kinds of mathematics. Jigs have mainly two functions during a specific stage of the working process: to hold the work in a defined position and to guide the tool in use. Usually, it is not possible to make jigs in a routine way, because each new jig is planned for a certain purpose and it requires a solution just for this unique purpose. The jig must be created. These situations were identified as typical problem solving situations in the cabinetmakers work and they assured that *"Almost in every project, at least one, more than one [jig]... and this is really one way to make art" (Jacob*). The following quotation belongs to the fieldnotes taken at the workshop where Jacob was building a design dining table: *"The table-top is ready and now he needs to position the legs in a way that they should not interfere with sitters. In order to fix the legs to the table he needs to drill four holes (mortises) for the dovetails to match, since each leg has a dovetail shaped tenon"*. Hence, Jacob had to make four dovetail shaped mortises to fix the legs of the table and for that he needed the router. The mortise was not any simple orifice. Jacob had to make a hole in a defined angle and with all precisely defined mathematical parameters (width, length and depth), so the hole could support and hold the tenon of the leg. To drill just that hole, Jacob had to design a jig that allowed the router to stay in place and make the required characteristics' perforation. This made the situation mathematical and thus, the creation of the jig was as well mathematical. This example illustrates that there were at least two mathematical problem solving situations emerging at the same time, intertwined with each other. One is when Jacob had to give mathematical attributes to the hole. The other one is when, in order to get this hole done, he needed to create a jig that allowed him to make this exact hole.

The efficiency of the problem solving process is not always determined by the cabinetmaker's advanced mathematics' skills. Then again, Anthony claimed that advance mathematical knowledge such as trigonometry might economize time and effort since the exact measurements can be established without the delay of trial and error or estimation.

In the analysis, more or less separate steps were identified in these problem solving situations. The participants told that they first approached the problem by looking for past experiences, either from themselves or from their workmates and put to the test different trials and modifications in practice.

"We try to find and remember old projects where we have been with the same kind of the problems and then we put a little bit of extra on top of that"… "Then probably you will negotiate with your friends who have been in that kind of situations before" (Thomas).

Thomas described how a problem solving situation was often shared and discussed with colleagues and generally it required critical thinking, experience and patience. They also tried to visualize the situation to find the solution. Thomas explained that sometimes he stayed awake at night, merely because finding mathematical solutions to problems fascinated him. When the problem was not solved in this way, the cabinetmakers ceased the conscious trying and thinking of the problem for a while: "*then you sit down and have a cup of coffee".* The solution to the problem might appear *"like a bolt out of the blue"* – even in a very different context where the problem was not consciously kept in mind. When the solution was found, the last and ultimate step was to put the solution into practice. Then the cabinetmakers confirmed the details and assessed the feasibility, practicality and quality of results "*and then at the end, when you have solved the problem, the only thing left to just cut, sand and finalize the surface"* (Thomas).

Problem solving as a creative process

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The steps identified in the cabinetmakers' work-based and practical problem solving process in the jig creation are to a considerable extent analogous to stages in the model of creative process developed by Jaques Hadamard (1945) decades ago. As a scientist, he was interested in mathematical inventions and, based on these, he developed a model for the process of invention. Later, it has been widely applied to model various creative activities. In Figure 1, the process of cabinetmakers' problem solving is drawn together with the creative process as modelled by Hadamard. All four steps of the problem solving process identified in the data flowed along with the creative process in a synchronized manner.

In Hadamard's model, the first stage "initiation" is featured by drawing on one's personal experiences and conscious, goal-oriented working. This is almost exactly what the cabinetmakers did since they first turn to their own or their workmates experiences to find the solution, as explicated in the previous chapter. These findings are consistent with the findings of Liljedahl (2009), where a group of mathematicians affirm that talking with colleagues is of great value when solving a problem, and in his work with both, pre-service teachers and mathematicians (2013) where the role of talking is as well an emergent theme of his data.

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Figure 1: According to our data, the stages of both the creative process and the problem solving process seem to be analogous.

FIGURE ONE APROX HERE!

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If the solution was not found, the cabinetmakers told they cease the conscious and intentional trying (the unconscious stages are marked with a discontinuous line). This second step relates to the "incubation" stage in Hadamard's model (1945). In particular, the flexibility and creative manner found in the cabinetmakers when facing the two first stages of the problem solving situation, namely the creation of a jig, is consistent with research that suggests that the use of mathematics at work is divergent from the mathematics taught at school (Gainsburg 2006, Noss, Hoyles & Pozzi, 2002).

Finally, the analysis showed the stage where the eruption of the visualization of a possible solution brings the cabinetmakers the idea and tools to proceed with the task. The third stage of finding the solution (illumination stage) is labelled with a sudden conscious insight "AHA-experience" and it is often loaded with affective aspects of the experience (Liljedahl 2013). This suggests why Thomas expressed excitement and enthusiasm and claiming to *love* spending time creating a solution for a problem. Both Thomas and Jacob described how, at a certain moment during a problem solving process, they were able to visualize the solution. According to them, the visualization was an image that often they would sketch and save as soon as it appeared. Jacob showed several sketches of projects and pieces he had visualized. The last step is labelled as verification stage where the solution will be tested and put into the use. In this stage, the visualization sketches were of great value to proceed in terms of accuracy and perfection.

As shown in Figure 1, this study suggests that the first two stages of the problem solving process overlap, due to the fact that they are not clearly separated stages and may occur at the same time. The participating cabinetmakers considered that in jig creation the most crucial phase was to conceptualize and to think, visualize and consider what the purpose of that particular jig was. The following extracts from the data refer to the initiation and incubation stages of Hadamard's creative process model and illustrate the need of thinking and planning:
"Making jigs is a simple thing. Thinking up jigs is the problem. How do you think them up, not how do you make them!" (Thomas)
"Maybe there are manuals, but I think every time you have to plan it and think about how to do it. First of all, what you want to do, what you are going to do and then you'll plan it. There cannot be examples for every situation, never" (Frank)

Furthermore, the cabinetmakers exhibit a great deal of flexibility during the process and in all the stages, while applying different methods and trials to try to visualize a possible solution to the problems. Often, they discuss with other cabinetmakers and share experiences to try to find a path. Along with the findings of Taylor regarding the creative process (2012), time becomes a key element, since it stretches and it is completely different for each process. For both, our participants as well as for Taylor, time is an unrestricted factor that characterises the processes. Unfortunately, time as a factor is not reflected Figure 1.

In research literature, the terms "creative process" and "problem solving process" have often been interchanged and used as synonyms most likely because of their similar characteristics, attributes and stages (see Leikin & Pitta-Pantazi, 2013; Csapó & Funke, 2017; Lubart, 2001) as illustrated in Figure 1. This study considers the concepts to be intertwined, but as they describe different phenomena they should be differentiated. The next section presents the modified version of Figure 1 to illustrate how the different stages of both processes based on the data are corresponding.

Discussion

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The findings concerning the mathematics cabinetmakers identified and used in their work are in line with many other previous studies about workplace mathematics (Williams & Wake 2007; Hoyles et al. 2001; Riall & Burghes 2000). The mathematics they used was in most cases very basic. Interestingly, the cabinetmakers also used mathematics (e.g. measurements and transformations) without self-evidently labelling it as mathematics. Even though the cabinetmakers identified many areas of mathematics that may be used and would be useful in their daily work, they used mathematics only if they were able to. Here, the cabinetmakers' different mathematical skills and knowledge are put to the limit. Where one of them was able to use trigonometry, and found it absolutely essential in cabinetmaking, the other thought that it was possible to manage without trigonometry but, admitted needing the help of his colleagues in this respect. The level of cabinetmakers' mathematical skills and knowledge seemed to restrict the choice of projects they could accept to complete. However, it must keep in mind that although the mathematics had a significant role in cabinetmakers' work and helped them to complete more demanding projects, the mathematics as such can never substitute the skilful craftsmanship with the wood.

A remarkable finding in this study was that, for cabinetmakers, the most common problem solving situation is making the needed jigs where mathematics was inevitably intertwined. The findings about problem-solving situations led to further ponder about the linkages between problem solving and creative processes. In this case of cabinetmakers, it is noteworthy that one creative process typically included several problem-solving processes (building jigs). The stages of problem-solving process and creative process share many similarities, yet, as our data reflected and according to Wimmer (2016) they should not be considered as identical processes. Figure 2 illustrates the similarities and the differences between the processes. During both, the creative process and the problem solving process, the goal is to find, to conceive a final product or solution. However, in the creative process one of the main traits of the final product (e.g. a dining table) or its attributes must be novelty or innovation. Sometimes this novelty has a gradation and may be a mere improvement of a previous product what defines the creativity. On the contrary, in the problem solving process, what matters is the viability of the solution (as it is in the case of jigs). In other words, novelty is a condition of possibility in the creative process as feasibility and practicality are for the problem solving one.

FIGURE 2 APPROXIMATELY HERE

Figure 2: Problem solving process and creative process based on our data

Having said that, depending on its level of novelty and innovation (see figure 2), the solution of problem solving may or may not be creative. According to the data, when the cabinetmakers create a jig, the aim is to create something with a purpose and its value depends on its usefulness and not on its novelty (i.e. can the jig hold the piece of wood in the needed position and does it give it room for modifying a specific angle or not). Therefore, the creation of a jig is a problem solving situation and it is not regarded by the cabinetmakers as a creative process, since the jig is meant to (at the same time) serve a definite purpose. For example, when a cabinetmaker wants to design a table, during the process he must invent and build several jigs to be able to make concrete cuts on the timber. These could be considered creative processes but no value is given to them for their uniqueness or originality. Their value is given for their suitability, and therefore, they are problem solving situations within the creative process of designing a table.

In figure 2, the solution of the problem solving process is located outside the process box since what is unknown is the procedure and gap between the departure point and the end result. A problem solving process may lead to a creative solution or to a less innovative one, but the validity of the solution does not depend of the level of creativity. On the contrary, in a creative process if there is neither innovation nor novelty, there is no creativity.

The findings and conclusions in this study are based on specific data in a specific context. In the future, more research and various data are needed to elaborate the conceptual and practical differences as well as the relationships between creative processes and problem-solving processes. This is the aim in the next stage of this project. The findings reveal that cabinetmakers constantly face problem solving situations along with the creative processes. Although there were no totally unexpected problems in the data, many of those problems were unique and had a number of unknown features. Hence, the cabinetmakers had very little prior experience of them. Being able to use more advanced mathematics helped them to solve those problems more efficiently, without wasting time and materials. This study suggests that the combination of craftsmanship, creativity, and efficient problem solving skills together with more than basic mathematical knowledge will help cabinetmakers in adapting and surviving in the future unstable labour markets. Even the future cabinetmakers should be able to create something beyond the capacity of machines.

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